JX-003-001515

Seat No. ______

B. Sc. (Sem. V) (CBCS) Examination

October - 2019

Mathematics : MATH-503-A

(Discrete Mathematics & Complex Analysis - I)

Faculty Code : 003
Subject Code : 001515

Time : **2:30** Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Numbers written to the right indicate full marks of the question.

1 Attempt all the questions : **20**

(1) Define equivalence relation.

(2) Define POSET.

(3) Write the least element and greatest element of POSET $(\{1, 2, 3, 4, 5, 6\}, /)$.

(4) Define equivalence class.

(5) State principle of duality.

(6) Draw Hasse Diagram for POSET $(P(\{a, b, c\}), \subseteq)$.

(7) Give an example of bounded lattice which is not complemented lattice.

(8) Write two properties of Boolean algebra.

(9) Write atom's of Boolean algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(10) Define Product of sum canonical.

(11) Find $\lim\limits_{z\to\infty} \dfrac{3z+5}{z+1}$.

(12) State C-R condition in Polar-Form.

(13) Define Analytic Function.

(14) Write condition for function to be Harmonic.

(15) State Milne Thomson Theorem.

(16) Define complex function?

(17) Define differentiability of a complex function $f(z)$.

(18) Define Analytic function.

(19) Define Entire function.

(20) What is Laplace equation for a complex function in polar form ?

2 (a) Attempt any **three** : **6**

(1) Define :

(i) Antisymmetric relation

(ii) Chain.

(2) Define Lattice as an algebraic system.

(3) Draw the Hasse diagram of (S_{120}, D).

(4) In usual notation prove that $A(x) = A - A(x')$.

(5) State and prove Isotonicity property.

(6) If (P, R) is a POSET then prove that (P, R^{-1}) is also a POSET.

(b) Attempt any **three** : **9**

(1) State and prove cancellation law for distributive lattice.

(2) If $(L, \leq)$ be a lattice in which $*$ and $\oplus$ denote meet and join respectively then prove that for $a, b \in L, a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

(3) Define Lattice and show that (S_{30}, D) is a Lattice or not.

(4) State and prove Modular inequality.

(5) State and prove De Morgan's law for Boolean algebra.

(6) Show that the lattice with three elements is a POSET.

(c) Attempt any **two** : **10**

(1) Prove that the direct product of two distributive lattice is distributive.

(2) State and prove distributive inequality for lattice.

(3) State and prove Stone's representation theorem.

(4) Prove that every finite lattice is complete.

(5) Define Boolean algebra and show that $(S_6, *, \oplus, ', 0, 1)$ is a Boolean algebra.

3 (a) Attempt any **three** : **6**

(1) Show that $f(z) = \overline{z}$ is not analytic.

(2) Find real and imaginary part of function $f(z) = \cos z$.

(3) Define Harmonic function and Harmonic conjugate function.

(4) State Cauchy integral formula for derivative.

(5) Prove that $\exp(z)$ is analytic.

(6) State fundamental theorem of algebra.

(b) Attempt any **three** : **9**

(1) Prove that $\int_0^{1+i} \overline{z} dz = 1$.

(2) Prove that $f(z) = e^y(\cos x + i \sin x)$ is not an analytic function.

(3) Prove that $f(z) = e^z$ is an analytic function and hence prove that $f'(z) = f(z)$.

(4) Show that $f(z) = z^2$ is an analytic and entire function.

(5) Evaluate : $\int_C \frac{(z^2+3)}{z^2(z-4)} dz$; where $C : |z| = 1$.

(6) State and prove Liouville's theorem.

(c) Attempt any **two** : **10**

(1) State and prove Morera's theorem.

(2) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y$.

(3) State and prove Cauchy integral formula.

(4) State and prove Cauchy's fundamental theorem.

(5) Find $\int_C \frac{z}{(2z-1)(z+1)} dz, \ C : |z| = 2$.
